Understanding the Physics of Launching Rings

In-depth look at how projectiles move

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Important Things to Note

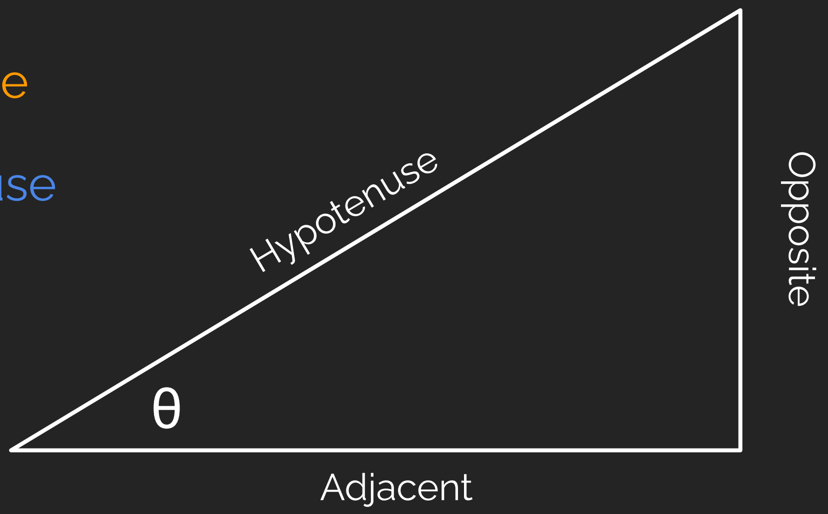
SOH, CAH, TOA

SOH → Sin: Sin(θ) = Opposite/Hypotenuse

CAH → Cos: Cos(θ) = Adjacent/Hypotenuse

TOA → Tan: Tan(θ) = Opposite/Adjacent

θ → Theta (symbol for angle)



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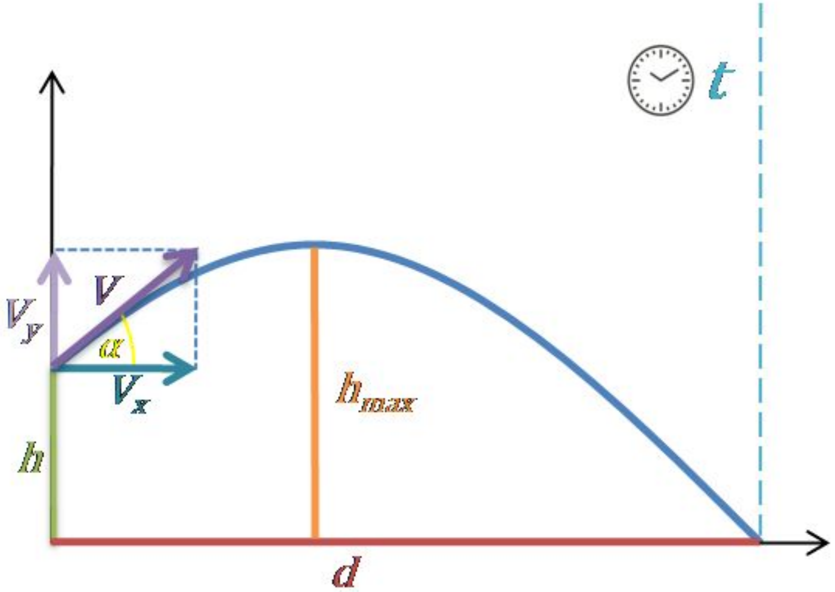
Overview Of Entire Trajectory



V – velocity
V_x – horizontal velocity
V_y – vertical velocity
α – angle of launch
h – initial height
t – time of flight
d – distance (range)
h_{max} – maximum height

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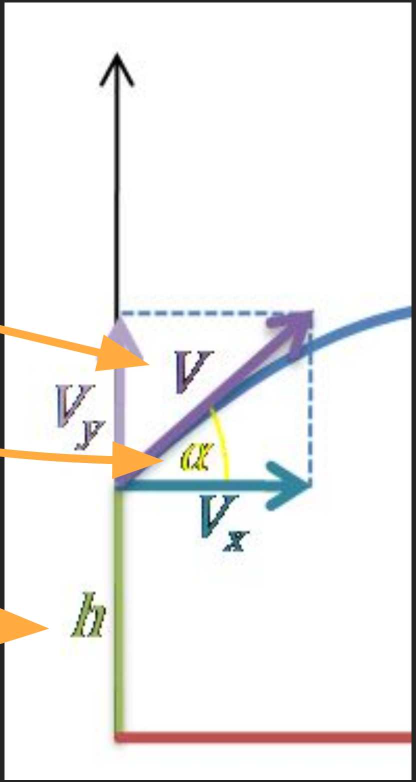
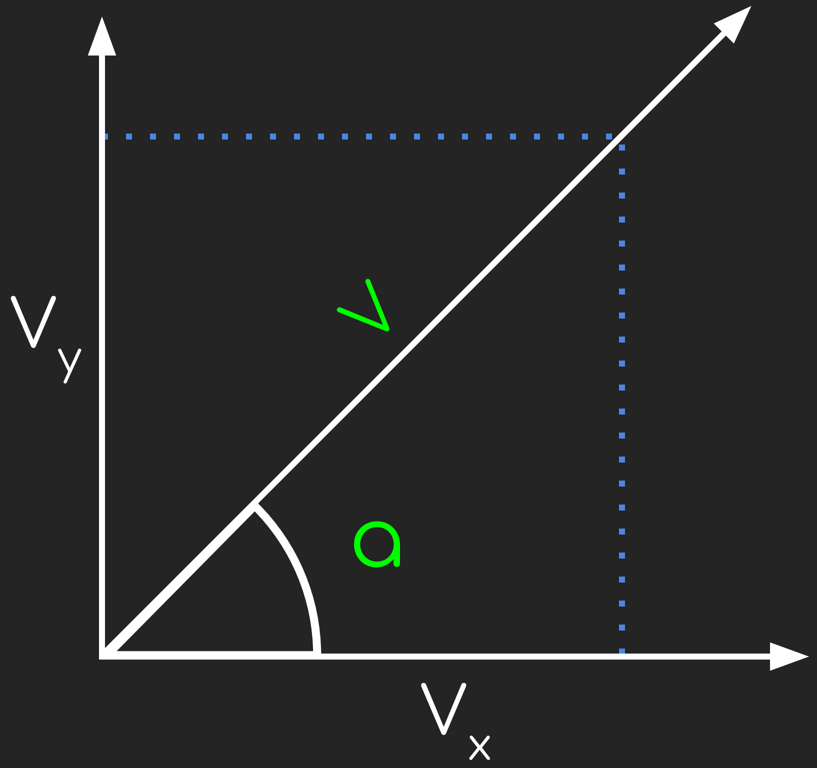
What we Already Know

- Velocity (V)
- Angle (a)
- Height from Ground (h)



Deriving Velocity Components

Calculating Velocity Components (Vertical/Horizontal)



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Calculating Horizontal Velocity Component

By creating a triangle using the dotted blue line, we can easily find V_x by using **cos**.

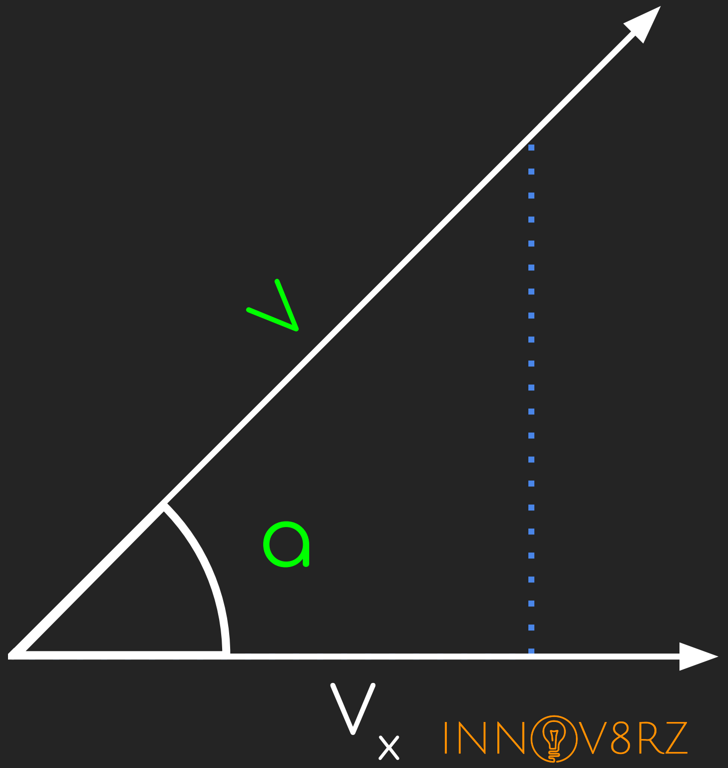
We know: V (hypotenuse) and the angle.

$\cos(\theta)$ = Adjacent/Hypotenuse

Multiply both sides by Hypotenuse:

$\cos(\theta)$ * Hypotenuse(V) = Adjacent (V_x)

Hence, $V_x = V * \cos(a)$



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Calculating Vertical Velocity Component

By creating a triangle using the dotted blue line and transferring angle ɑ from the previous triangle, we can easily find V_y by using **sin**.

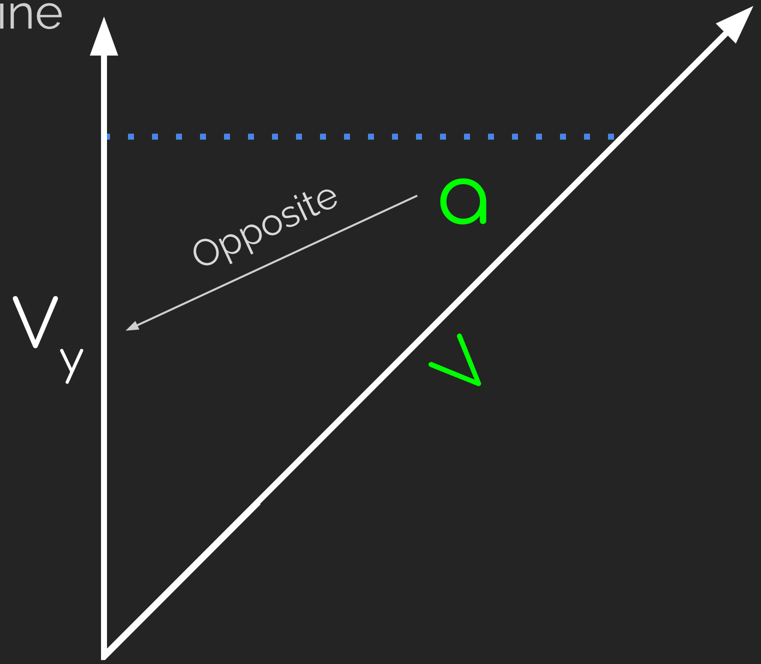
We know: V (hypotenuse) and the angle.

$\sin(\theta) = \text{Opposite}/\text{Hypotenuse}$

Multiply both sides by Hypotenuse:

$\sin(\theta) * \text{Hypotenuse}(V) = \text{Opposite}\ (V_y)$

Hence, $V_y = V * \sin(\alpha)$



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Distance Equations

Distance, Rate, Time → Horizontal Distance

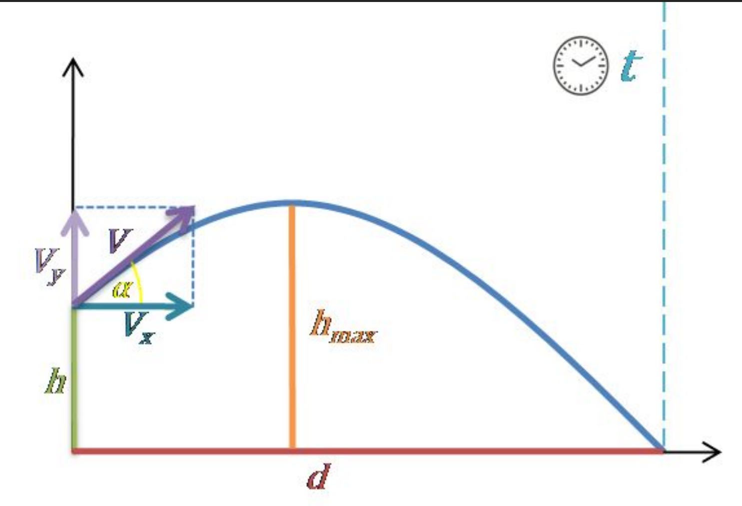
Distance = Rate * Time

x → Horizontal Distance

V_x → Horizontal Velocity

t → Time

→ **$x = V_x * t$**



Vertical Height

If NO Gravity:

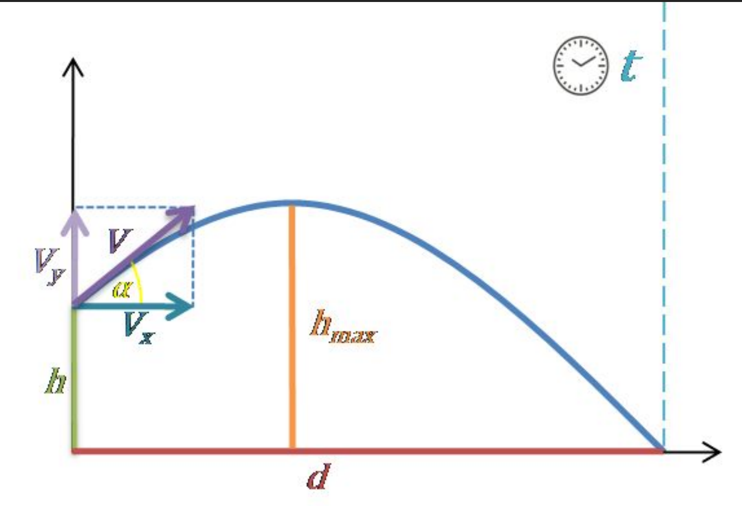
$y = h + (V_y * t)$

y will increase as time increases (if V_y is > 0)

If Gravity:

$y = h + (V_y * t)$ **$- (\frac{1}{2} * gt^2)$**

g → Gravitational Acceleration (9.8 m/s/s or 9/8 m/s²)



Calculating Time of Flight

At what time the ring hits the ground

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Time of Flight

We can use our expression from calculating Vertical height and set it to 0, because that is when it hits the ground:

$h + V_y * t - g * t^2 / 2 = 0$

Now, using the quadratic formula, we can reorder to get:

$-\frac{1}{2} g * t^2 + V_y * t + h$ where $a = -\frac{1}{2} g$, $b = V_y$, and $c = h$

Then, we use those variables and plug them into the quadratic formula to solve for t

$t = [V * \sin(a) + \sqrt{(V * \sin(a))^2 + 2 * g * h}] / g$

Calculating Distance

How far the ring hits the ground from the start
(Projectile Range)

Range of Ring

Formula for Range → $R = V_x * t$

$t = [V * \sin(a) + \sqrt{((V * \sin(a))^2 + 2 * g * h)}] / g$ ← from time of flight calculation

$V_x = V * \cos(a)$

Final Formula: $\mathbf{R = V * \cos(a) * [V * \sin(a) + \sqrt{((V * \sin(a))^2 + 2 * g * h)}] / g}$

Above, we substituted $\mathbf{V_x}$ for "V * cos(a)" and **t** for the equation from before

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Calculating Max Height

Apex of the Ring's Trajectory

Max Height

What is Max Height? Max height is when the ring stops increasing height, and when V_y is 0, meaning there is no vertical velocity. This is the apex or vertex of our height vs. time graph.

From: **$h + V_y * t - ½ * (g * t^2)$**, we can convert to quadratic form ($ax^2 + bx + c$):

$-½ g * t^2 + V_y * t + h$ where $a = -½ g$, $b = V_y$, and $c = h$

To find the vertex, we use the vertex formula: **$x = -b/2a$**, x in our graph is t

So, time at which height is most (or V_y is 0) is when: $t = -(V_y)/2*(-½g)$ → **$t = V_y / g$**

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Max Height

Now, we take our original formula from not so long ago: $V_y * t - ½ g * t^2$

But now, **$t = V_y / g$**, so we substitute t to get:

$h_{max} = V_y * (V_y / g) - ½ g * (V_y / g)^2 = V_y^2 / g - ½ V_y^2 / g = ½ V_y^2 / g = \mathbf{V_y^2 / 2 * g}$

We now substitute $V * \sin(\alpha)$ for $V_y \rightarrow h_{max} = \mathbf{(V * \sin(\alpha))^2 / 2 * g}$

Since h just shifts everything up, we can just add h to the equation:

$h_{max} = h + (V * \sin(\alpha))^2 / 2 * g$

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YAY!

Now, we have calculated all of the equations!

Quick Recap of All Equations

- Horizontal velocity component: $V_x = V * \cos(a)$
- Vertical velocity component: $V_y = V * \sin(a)$
- Time of Flight: $t = [V * \sin(a) + \sqrt{((V * \sin(a))^2 + 2 * g * h)}] / g$
- Range of Ring: $R = V * \cos(a) * [V * \sin(a) + \sqrt{((V * \sin(a))^2 + 2 * g * h)}] / g$
- Maximum Height: $h_{max} = h + (V * \sin(a))^2 / 2 * g$

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